Mutual Synchronization of Passive Hydrogen Masers

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***Abstract*—In the experiment frequencies of two passive hydrogen masers are mutually adjusted by means of PI-servo loop which obtains error value from the frequency comparator and controls synthesizer codes of the masers. As a result, output signals of both masers have got the same mean frequency and frequency stability is improved for the averaging times more than the control loop time constant value. It is also shown that summary signal of two mutually synchronized (and adjusted by phase) masers has better short-term stability and lower phase noise spectrum than the separate maser signals have. Simple theoretical and simulation analysis has been done to retrieve the conditions of frequency stability improvement or degradation in ensemble of two coupled clocks.**

Keywords—synchronization; hydrogen maser; frequency stability; phase noise; coherent combining

I. Introduction

It is well known idea to exploit uncorrelatedness of noises of separate atomic clocks to improve stability of the atomic clock ensemble [1]. Usually this idea is implemented as a paper clock algorithm. Sometimes real signals are steered according to the paper clock algorithm and improved timescale is generated. It is also known that auxiliary oscillator can be locked to the averaged frequency of several sources and its frequency stability can be better than the stabilities of the input signals [2]. The same improvement should happen in case of mutual synchronization of atomic clocks or any oscillators with close to each other characteristics, but there is a fear that mutually locked oscillators in fly-wheel mode can be unstable. The experiment with two passive hydrogen masers, simple theoretical analysis and modeling has been done to clarify the effects of mutual coupling on frequency stability of clock ensemble.

Apart from possible time keeping application, coherent combining of signals of mutually synchronized masers allows improving short-term stability and phase noise characteristics. In case of active hydrogen masers, their short-term stability is important for radio astronomy measurements (VLBI) and for reference signal generation in fountain clocks.

II. Theory

Frequency of an oscillator (clock) is considered to be a control parameter. Some external circuit implements coupling of oscillators. Linear discrete in time phase model of two coupled oscillators can be described by the following equations:

$$\begin{cases} \mathbf{X}_{1,n+1} = \mathbf{F}\mathbf{X}_{1,n} + \mathbf{w}_{1,n} + \mathbf{B}u_{12,n}, \\ \mathbf{X}_{2,n+1} = \mathbf{F}\mathbf{X}_{2,n} + \mathbf{w}_{2,n} + \mathbf{B}u_{21,n}, \end{cases} \tag{1}$$

$$\mathbf{X}_n = \begin{bmatrix} x_n \\ y_n \end{bmatrix},\ \mathbf{F} = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix},\ \mathbf{B} = \begin{bmatrix} \Delta t \\ 1 \end{bmatrix}, \tag{2}$$

$$\mathbf{w}_n = \begin{pmatrix} \xi_n \\ \eta_n \end{pmatrix} \sim N(0, \mathbf{Q}_n), \tag{3}$$

where x_n, y_n are components of state vector related to oscillator phase and frequency respectively, Δt is a steering interval, $\mathbf{w}_n$ is a normal two-dimensional noise with covariance matrix $\mathbf{Q}_n$. For PID-controller linear control actions can be written as follows:

$$u_{pq,n} = -g_x \hat{x}_{pq,n} - g_y \hat{y}_{pq,n} - g_d (\hat{y}_{pq,n} - \hat{y}_{pq,n-1}), \tag{4}$$

where $\hat{x}_{pq,n}$, $\hat{y}_{pq,n}$ are measured phase and frequency differences between clocks p and q; p, q = 1, 2; g_x, g_y, g_d – controller coefficients.

In the experiment described in the next section phase differences are measured by comparator and used for frequency difference estimation. Obtained phase and frequency differences are:

$$\hat{x}_{pq,n} = x_{p,n} - x_{q,n} + \varepsilon_{pq,n},\ \hat{y}_{pq,n} = \frac{1}{\Delta t}\left(\hat{x}_{pq,n} - \hat{x}_{pq,n-1}\right), \tag{5}$$

where measurement noise $\varepsilon_n \sim N(0, \sigma^2_{\varepsilon,pq})$.

Analyzing synchronization phenomenon of coupled oscillators it is usual approach to rewrite equations in the variables of difference and sum. Equations for phase and frequency differences describe synchronization of the oscillators. We assume that gain coefficients belong to the region of stability of stationary solution. Frequency adjustment is achieved if $g_y \neq 0$. Phase adjustment is achieved if additionally $g_x \neq 0$. Due to the symmetrical coupling (controller coefficients are the same for both oscillators), synchronization should occur at the center (average) frequency. Therefore, frequency fluctuations of the oscillators are also averaged, and frequency stability improvement can be expected.

However, it is important to check the equations for summary variables: $\Phi_n = x_{1,n} + x_{2,n}$, $\Omega_n = y_{1,n} + y_{2,n}$.

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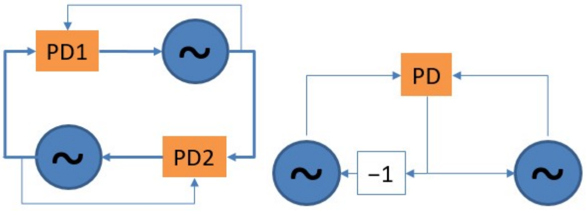

(a) (b)

Fig. 1. Ring coupling of oscillators with phase locked loops (a), mutual coupling using common phase detector (b).

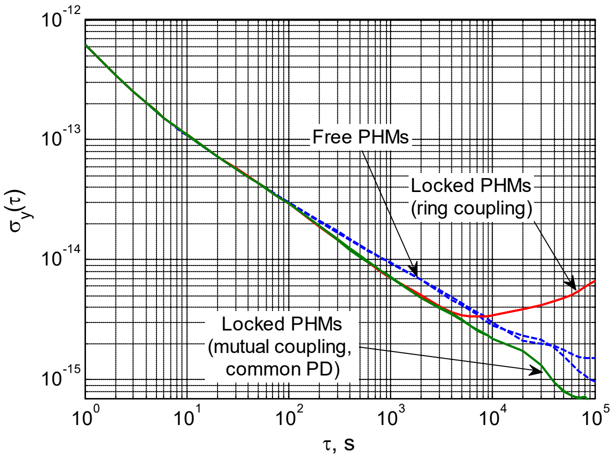

Fig. 2. Results of modeling for two schemes of coupling (Fig. 1).

$$\Phi_{n+1} = \Phi_n + \Omega_n \Delta t + \xi_{1,n} + \xi_{2,n} + (u_{12,n} + u_{21,n})\Delta t,$$
$$\Omega_{n+1} = \Omega_n + \eta_{1,n} + \eta_{2,n} + (u_{12,n} + u_{21,n}). \quad (6)$$

Equations (6) tell that only if control actions are absolutely symmetrical: $u_{12,n} = -u_{21,n}$, then summary phase and frequency fluctuations of coupled oscillators will be the same as the sums of fluctuations of uncoupled oscillators. Otherwise additional random walk frequency noise will be generated in the ensemble of mutually synchronized oscillators.

For the considered mathematical model (1) – (5), symmetry breaking of control actions can happen if two different comparators are used to measure phase differences $\hat{x}_{12,n}$ and $\hat{x}_{21,n}$, so measurement noises in comparators are uncorrelated $\varepsilon_{12,n} \neq -\varepsilon_{21,n}$, and consequently $u_{12,n} \neq -u_{21,n}$. Two phase locked loops coupled in a ring in Fig. 1a illustrate this case. If one common comparator (PD) is used (Fig. 1b) no extra random walk frequency noise to be generated.

Both coupling schemes were simulated numerically (Fig. 2) [3]. System parameters used in the modeling: g_x=0, g_y=0.05, g_d=0.01, σ_ε=10^{-14}. Frequency drifts of the oscillators were equal to 0. Frequency control actions are rounded to the discrete values with the step 10^{-15} (related to passive hydrogen masers VCH-1008 and CH1-1007). Time constant for these controller parameters is around 100 seconds. Frequency stability of locked oscillators is improving for averaging times more than 100 s, but even small uncorrelated measurement noise (σ_ε=10^{-14}) leads to the degradation of Allan deviation at long term for the oscillators coupled in a ring.

Even if the scheme with common PD (Fig. 1b) is used control actions can't be identical by absolute value, small noise of controller should be taken into account, so $u_{12,n} \neq -u_{21,n}$.

It has been verified by modeling that small delays in the control loop (less than time constant) do not lead to degradation of frequency stability at long term. Therefore, unavoidable variations and bias in control actions seem to be the only factor, which leads to degradation of long-term frequency stability of mutually coupled oscillators.

III. Experiment

The experimental setup is shown in Fig. 3. Two passive hydrogen masers (PHM) CH1-1007 are measured by frequency comparator VCH-314. Frequency difference counts with time interval 1 s are processed in the computer (PC) which generates frequency code corrections for both masers every 10 s. Proportional-integral controller with time constant around 100 s is implemented. Output 5 MHz signals of the passive masers are measured with respect to active hydrogen maser CH1-1033 by multichannel frequency comparator VCH-315. Additionally, output signals of the passive masers were roughly adjusted by phase and combined in ZFSC-2-1W-S+. Summary signal is also measured by VCH-315 with respect to the active maser signal.

Fig. 4 shows dynamics of frequencies of free and mutually locked PHMs. Note that: frequency noise has been reduced due to mutual synchronization, PHM frequencies has been adjusted to the center frequency, but the frequency drift of the coupled PHMs does not equal to the average drift of the uncoupled PHMs. This effect can be explained by a very small bias in control actions: $u_{\text{applied}} = u_{\text{calculated}} + \delta u$.

Allan deviation plots for passive masers with switched on/off coupling are presented in Fig. 5 (linear frequency drift is removed). Duration of the experiment with synchronization is 24 days. ADEV of mutually locked masers is considerably less than ADEV of the same masers without control for the averaging times much greater than 100 s. Summary signal has better short-term stability than the separate signals have.

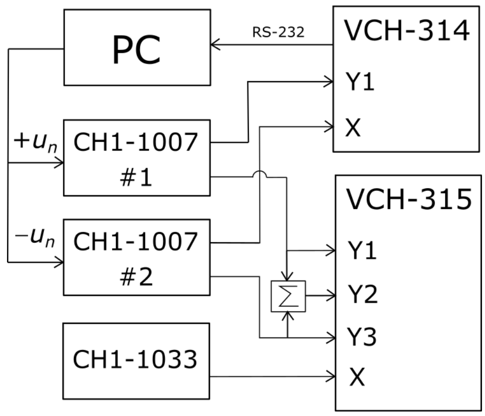

Fig. 3. Experimental setup.

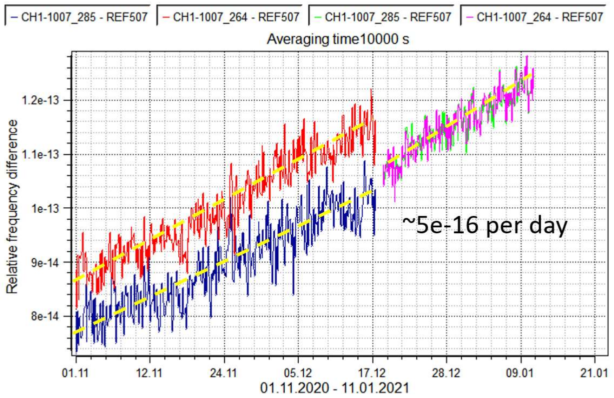

Fig. 4. Frequencies of PHMs before and after coupling.

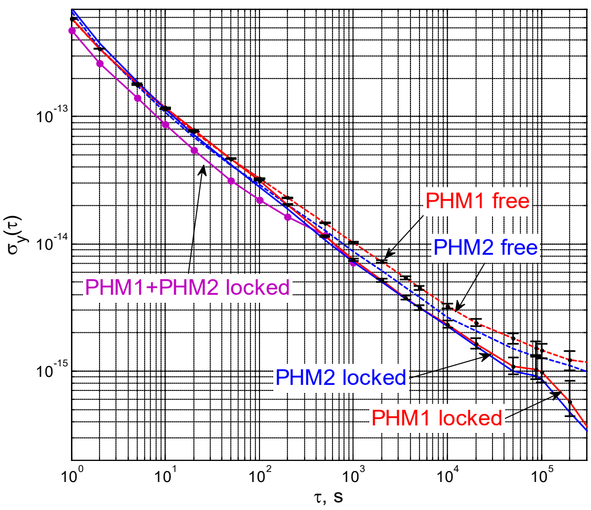

Fig. 5. Allan deviations experimatally obtained for free PHMs, mutually synchronized PHMs and summary signal of mutually synchronized PHMs.

The experiment was repeated several times for different pairs of PHMs. In all experiments frequency stability was improved at middle term for averaging times more than 100 s. In some experiments, there was no clear observation of improvement or degradation of frequency stability at long term (for averaging times more than one day).

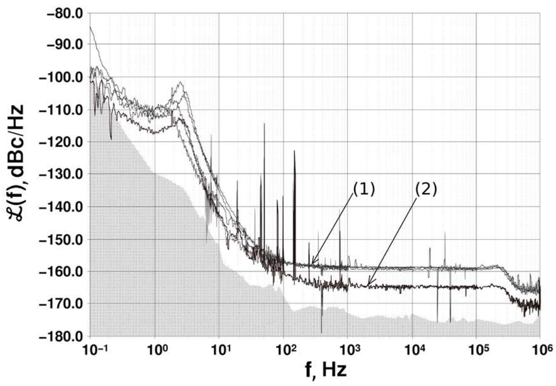

Fig. 6. Power spectral density of separate PHM signals (1) and summary signal of four PHMs adjusted by phase and frequency (2).

Fig. 6 shows the result of coherent combining of four PHMs, mutually synchronized and manually adjusted by phase. Phase noise reduction of summary signal is clearly observed.

IV. Conclusions

Theoretical analysis and modeling results tell that mutual coupling of clocks should be done very accurately. Different measurement noise, variations and biases in control actions for synchronized clocks lead to the degradation of long-term frequency stability. Since it seems impossible to absolutely exclude variations in control actions, mutually synchronized clocks can't be used for time keeping. However, if steering to stable reference can be done, then mutually synchronized clocks can improve middle term frequency stability and allow coherent combining to improve short-term stability.

Experimental results approve that at least middle-term frequency stability of mutually synchronized clocks is improved, short-term stability and phase noise are reduced for coherently combined signals.

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